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Fracture Mechanics of Non-Metallic Materials [and Discussion]

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Fracture mechanics of non-metallic materials

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An outline of linear elastic fracture mechanics (l.e.f.m.) is given with an emphasis on those aspects most relevant to non-metallic materials. Provided that the nonlinear zone of energy absorption surrounding the crack tip is small compared with other dimensions, then a K_c or G_c value may be used. A simple extension of this concept can include elastically nonlinear materials such as rubber. Examples of the use of this method are then given for polymers, rubber and wood, and include some discussion of the difficulties involving plane strain–plane stress transitions. The role of K_c as a characterizing parameter in time-dependent, fatigue and environmental behaviour is then described with several examples, and it is concluded that plane strain fractures may be achieved with a wide range of values for any material. The consequences of this in choosing a design criterion are then discussed.

INTRODUCTION

It would be easy to gain the impression that fracture mechanics only has utility in design with metals. Certainly, there are structures of extremely high criticality where metals are the only feasible materials and there is a wide range of less critical structures where they are a good economic option. However, there are many examples where the special properties of other materials make them preferable to metals so that they become important parts of structures. One may cite the high elasticity of rubber here and its use in tyres and suspensions, or the corrosion resistance of plastics as examples. For major load-bearing components, metals are still the dominant material but even here fibre composites are now being utilized. When the level of criticality is somewhat less, then non-metals become increasingly common and in particular glass fibre reinforced plastics and timber are widely used. Natural products such as wood and rubber have, perhaps, become unfashionable with increased synthetic skills, but they are very versatile and in high-cost energy economies may well become more widely used.

It is also worth considering the relative costs of highly critical, but small volume, components and those of mass-produced articles in which failure is often not very spectacular but nonetheless expensive. The design of plastic water and sewage pipes is an excellent example of the latter. Since large quantities are required and they must almost always be buried, there are very high installation costs, which can only be justified by long service lives. Failures of these pipes can thus lead to severe cost penalties as well as considerable inconvenience. In this, and in almost all other examples, it is the fracture of the component that causes most difficulty, and in most cases such fracture is by the growth of cracks. These may be internal and diffuse, as in composites, or a single crack which grows slowly and becomes unstable, leading to catastrophic failure. The growth of a cut in rubber, for example, which is a frequent cause of tyre failure, is a slow crack propagation phenomenon.

There is thus a practical need to understand the phenomena of crack growth in a wide range of materials and to incorporate such an understanding into design procedures. That there are differences in the behaviour of such widely differing materials as ceramics and rubber is obvious, and emphasizing them does not contribute to simplifying our picture of the phenomena. The key to successful designs is an understanding based on a sound theoretical structure and this is provided by fracture mechanics. It is not an understanding in itself but a framework in which many disparate effects may be viewed and understood, and we will examine the behaviour of several non-metallic materials in terms of the theory.

FRACTURE MECHANICS ANALYSIS

Linear elastic fracture mechanics, l.e.f.m., contains the basis of most of our understanding of fracture phenomena. It is not adequate for all fractures and, indeed, a good deal of this paper will be taken up with describing deviations from this simple basis. Such deviations arise from nonlinearity and irreversibility, which occur in many materials and are at variance with the linear elastic part of l.e.f.m., but the concepts embodied in the linear theory are capable of being extended to these more complicated cases.

It is often true that such phenomena are more easily understood as deviations from the linear model rather than attempting to formulate a general nonlinear theory. There have been attempts to establish such a theory but their value is obscured by extensive analytical difficulties, which can only be overcome in the linear model.

L.e.f.m. assumes that the material deforms according to the laws of classical elasticity theory in that it undergoes small strains and obeys Hooke's Law. The analysis describes the behaviour of a perfectly sharp crack in such a material when it is loaded. Two quite distinct approaches can be used in the analysis, as in all linear elastic materials. In the first, we may examine a conventional stress analysis of the crack and define the stress, σ_1, local to the crack tip such that

$$\sigma_1 = K/\sqrt{(2\pi r)}\, f(\theta), \tag{1}$$

where r is the distance from the crack tip, $f(\theta)$ is the angular distribution of stresses about the crack line such that $f(0) = 1$, and K is the stress intensity factor. The crucial point is that the form of the stress distribution local to the crack tip is the same for all loads and only its magnitude is changed by external factors, such as load and crack length through K. K is usually expressed in terms of the remote stress, σ, normal to the crack, and the crack length, a, such that

$$K^2 = Y^2\sigma^2 a, \tag{2}$$

where Y^2 is a dimensionless factor dependent on geometry and loading configuration. Since fracture is a local phenomenon and must occur in the region close to the crack tip, it is intuitively reasonable to consider a criterion

$$K = K_c \tag{3}$$

at fracture, since the stress state intensity is typified by K which remains finite in the crack tip region.

An entirely equivalent method for the linear elastic model is to consider the energy release rate, G, of a growing crack. The original fracture mechanics work of Griffith (1920) was based on such an approach and postulated a stability criterion for crack growth of

$$G \geqslant G_c. \tag{4}$$

It can be shown easily that the two conditions are identical and that

$$K_c^2 = EG_c. \tag{5}$$

In the original work, the analysis was directed at glass, where any form of nonlinearity is very slight and a completely elastic fracture is obviously very near to the truth. The use of such an analysis for materials where this is not so led to controversy, but the power of the method was established when it was realized that the analysis could be used when nonlinearity did occur. If a stress, σ_c, is defined at which linear behaviour ceases, then a zone size, r_c, may be defined from (1) such that

$$r_c = K^2/2\pi\sigma_c^2, \tag{6}$$

which defines the extent of nonlinear behaviour. If r_c is very much less than all the other dimensions of the body, then the elastic analysis and K still define the stress state, and K_c and G_c could be used to describe fracture (Irwin 1948; Orowan 1955). It is not necessary to describe the processes embodied in σ_c since they are lumped together in the gross fracture parameters. This is essentially the approach to be described here, in that specimens are designed to satisfy the size criterion so that a G_c or K_c can be found. Provided that the gross stress levels are not too high, then most materials are reasonably linear and the theory describes their behaviour. For linear but anisotropic materials, the appropriate Y^2 factors must be found and an equivalent to E must be computed for (5) (Sih *et al.* 1965).

For nonlinear elastic materials, the energy analysis may be performed in a similar way to the linear model and a G (which is usually written as T) may be computed, as given by Rivlin & Thomas (1953). Specially designed specimens are frequently employed to overcome calibration problems, although conventional tests can be used with strain-dependent factors (Greensmith 1963). An equivalent factor to K presents problems because of strain-dependent modulus and the concept is little used in work on rubber, for example. One is led here to assume a more fundamental basis for the energy criterion but this, while intuitively appealing, is largely a function of the conservation of energy in elastic systems.

The problem of which is the more fundamental criterion embodied in K and G in the linear analysis arises often in non-metallic materials when deviation from elastic behaviour occurs. For example, many polymers are linearly viscoelastic to a modest degree. This means that their behaviour is linear but time-dependent and contains dissipative effects. Energy-based analyses can be performed, but they are very problematical and a time-dependent characterizing parameter approach based on K is much simpler and appears to be adequate for practical purposes (Williams 1972).

When larger plastic flow occurs, the problems faced are the same as in metals and it appears that the contour integral, J_c, viewed as a characterizing parameter, is the best approach (Turner 1977). Indeed, it may be an indication of the relatively simple nature of non-metals problems, but there are few examples of where highly developed elastic–plastic analysis has been of value. Linear theory with the addition of very simple crack length plasticity corrections, coupled with plastic collapse analyses, has proved adequate (see, for example, Newmann & Williams 1980).

K_c AND G_c DETERMINATION IN NON-METALS

It would be expected that those materials that have a high stiffness and a low strength would conform most closely to l.e.f.m. theory, and this is confirmed by the behaviour of organic glasses such as polystyrene. Figure 1 shows a graph of $\sigma^2 Y^2$ against a^{-1} for some surface notches in this material (Parvin & Williams 1975) and the low degree of scatter and good linearity confirm the general utility of the analysis. Such an independence of K_c of crack length is an essential requirement if the theory is to be valid. Polystyrene is a rather brittle polymer under most practical conditions and is used mainly because of good clarity and low distortion behaviour. It is useful to use its toughness as a scale of reference since $K_c \approx 1\ \mathrm{MN/m^{\frac{3}{2}}}$ ($G_c \approx 350\ \mathrm{J/m^2}$). Figure 2 shows a similarly brittle material, a cross-linked thermosetting polyester frequently used as a matrix for glass fibre reinforced plastics. The basis of the plot is slightly different here (σY against $(\pi a)^{-\frac{1}{2}}$, π being removed from the calibration factor), but there is again excellent linearity giving $K_c = 0.6\ \mathrm{MN/m^{\frac{3}{2}}}$ ($G_c = 120\ \mathrm{J/m^2}$). Both of these materials are quoted without a rate since they are only very slightly viscoelastic. In other glassy polymers, there is much stronger rate dependence and here K_c (and G_c) are time dependent; for example, polymethylmethacrylate (PMMA) has a K_c varying from 0.7 to $1.7\ \mathrm{MN/m^{\frac{3}{2}}}$ (Marshall *et al.* 1974) depending on test condition and will be considered in the next section.

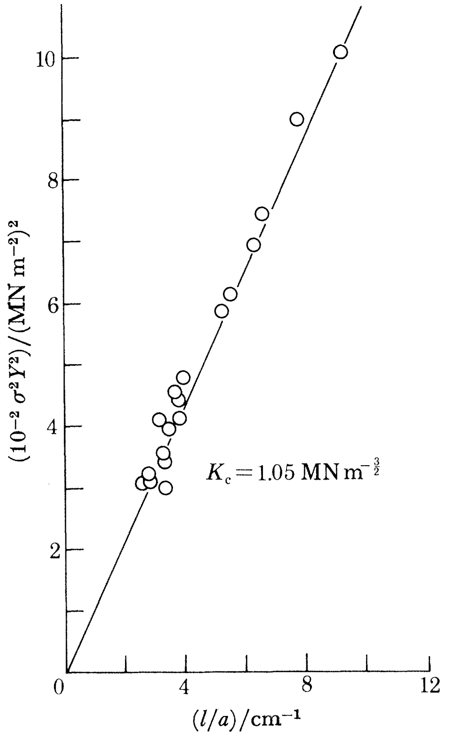

FIGURE 1. $\sigma^2 Y^2$ as a function of l/a for polystyrene at 20 °C, surface notches (Parvin & Williams 1975).

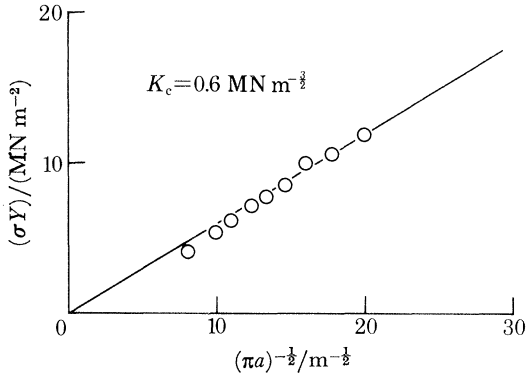

FIGURE 2. Fracture data for a polyester resin at 20 °C surface notches (data supplied by Dr G. P. Marshall).

The behaviour of a much tougher polymer is illustrated in figure 3, which shows σY against $a^{-\frac{1}{2}}$ for a polypropylene at -60 °C and a low rate (Fernando & Williams 1980). Here we have a plane strain fracture and the results are for four different testing configurations, namely single edge notched tension and bending, and surface notched tension and bending. Again, there is excellent linearity and also an independence of loading mode when the appropriate Y factor is used, providing further confirmation of the basic method. The toughness is much greater here at $4.7\ \mathrm{MN/m^{\frac{3}{2}}}$.

In anisotropic materials, there are severe difficulties with the method if the anisotropy influences the crack growth direction since the basic analysis is for a crack propagating in a self similar manner. If this restriction is met, however, then the Y^2 factors are essentially the same as the isotropic model and K_c values may be found. This condition restricts the results to effectively the 'along the grain' value which is usually the minimum for a material and therefore consistence with the lowest value l.e.f.m. concept. For aligned fibre composites, this value is usually not much different from the matrix value, as one might expect. The example shown in figure 4 is for wood, which may be regarded as an anisotropic composite, and here we have quite good linearity although there is much greater scatter in such a natural product (Ewing & Williams 1979). Here, we have $K_c = 0.47\ \mathrm{MN/m^{\frac{3}{2}}}$ and $G_c = 105\ \mathrm{J/m^2}$, and this is a reasonably representative value for a range of timbers (Barrett 1975; Schniewind & Pozniak 1971).

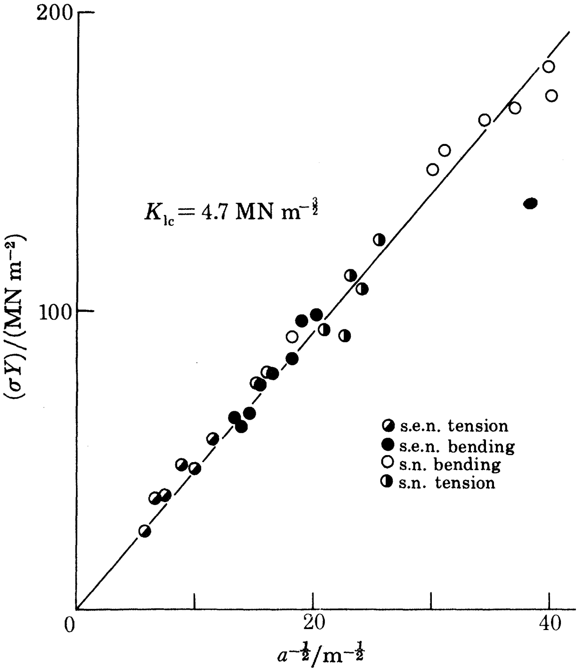

FIGURE 3. Fracture data for polypropylene at -60 °C for four test configurations (Fernando & Williams 1979).

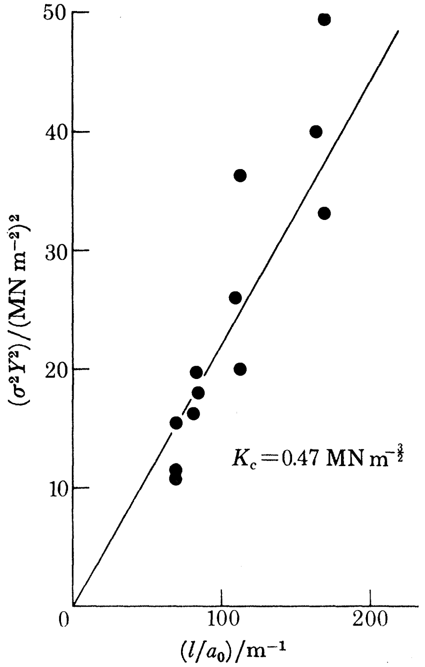

FIGURE 4. Fracture data for wood: Scots Pine in SEN at 20 °C (Ewing & Williams 1979).

Composites reinforced with randomly distributed fibres are usually described quite well if due regard is taken of the often quite large damaged zone that replaces the plastic zone. Figure 5 shows some data in bending on a chopped strand mat reinforced polyester resin tested in bending. There is more scatter here than on a homogeneous material, but a sensible $K_c = 7.0\ \mathrm{MN/m^{\frac{3}{2}}}$ can be deduced, showing that the addition of 33 % glass results in an order of magnitude increase in K_c over the matrix material. Another randomly reinforced composite is shown in figure 6, in this case chipboard, which is composed of wood fibres bonded by 10 % by mass phenol formaldehyde resin. Again, there is scatter but a K_c of $1.3\ \mathrm{MN/m^{\frac{3}{2}}}$ may be deduced.

In many non-metallic materials, their toughness in practice is not that of a plane strain value but is such that there is considerable energy absorption by shear yielding or other micromodes of deformation. A very good example of this is ABS (acrylonitrile butadiene styrene), a thermoplastic made by a combination of polystyrene and a rubber. The structure of the material is such that both shear yielding and a stable microvoiding process, known as crazing, can occur easily. When notched samples are loaded, there is considerable deformation evident and any form of plane strain failure is difficult to achieve. Observation of the crack tip during loading gives a crack initiation value and figure 7 shows some results for an ABS in both tension

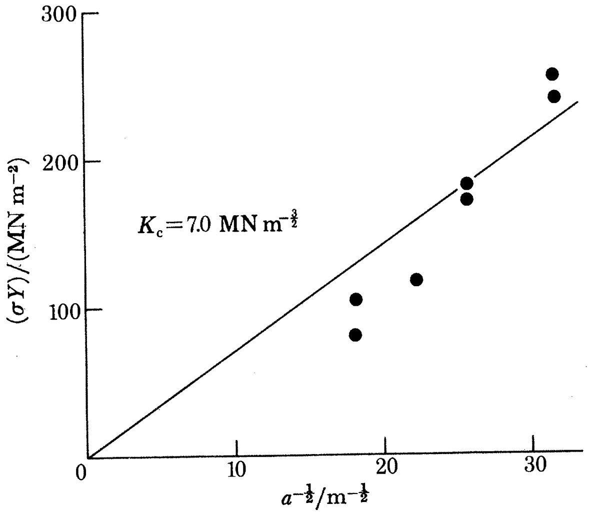

Figure 5. Fracture data on glass fibre reinforced polyester resin at 20 °C (data supplied by Dr G. P. Marshall).

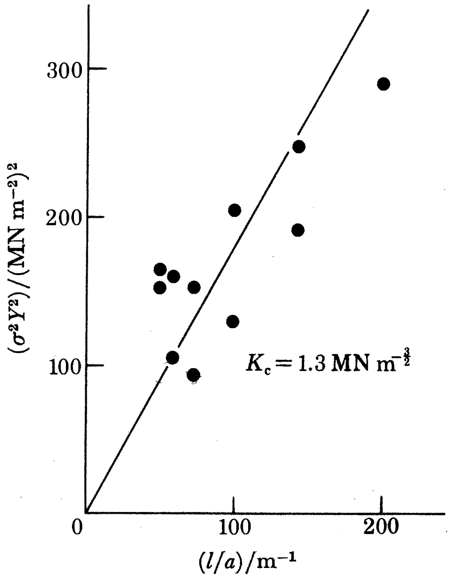

Figure 6. Fracture data on chipboard at 20 °C: wood fibres in phenol formaldehyde resin.

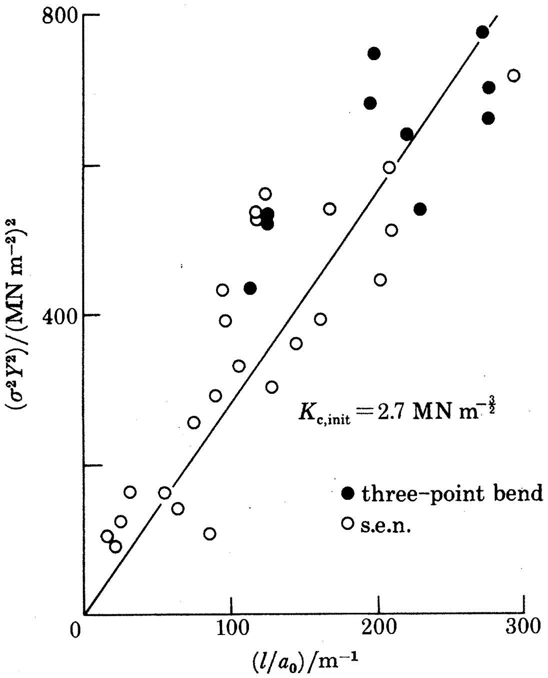

Figure 7. Fracture initiation data on ABS at 20 °C (Newmann & Williams 1980).

and bending, which give a single K_c value of 2.7 MN/m$^{\frac{3}{2}}$ and is probably a fair approximation to a plane strain value (the scatter is mainly attributable to the difficulty in observing initiation) (Newmann & Williams 1980). Considerable crack tip deformation takes place after initiation, however, and after a small amount of crack growth, there is final instability. The final load has been used to calculate the stresses shown in figure 8 and it is clear that the linear theory is not adequate here. Although the plastic zone is substantial, it does not constitute full yielding and the crack length may be corrected by a small length to account for this, as shown in figure 9,

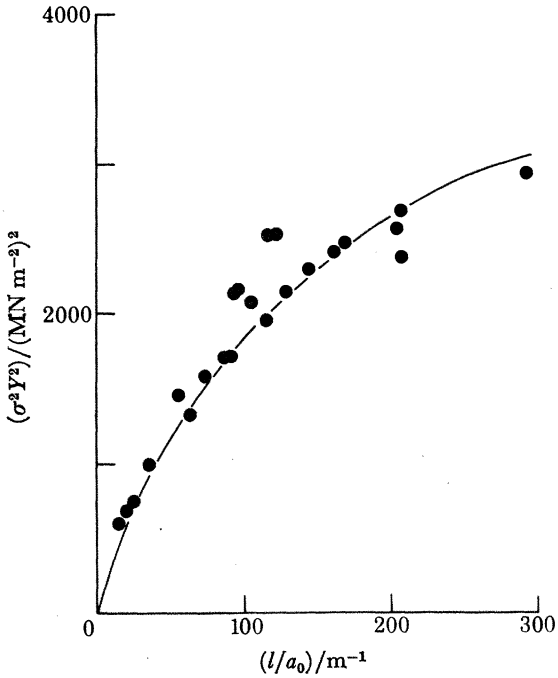

Figure 8. Fracture instability data in ABS at 20 °C: in tension (Newmann & Williams 1980).

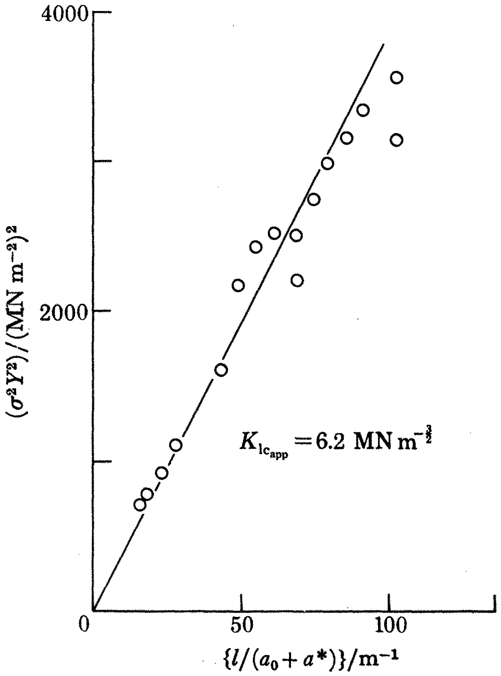

Figure 9. Fracture instability data in ABS at 20 °C: crack length corrected for plastic zone size (Newmann & Williams 1980).

where reasonable linearity is achieved with K_c = 6.2 MN/m$^{\frac{3}{2}}$. These are tension values, but for bending a lower number is measured by this method. Thus, for these large plastic flow materials, the apparent K_c values are not genuine in that they include a plastic flow contribution which is constraint governed and thus different in different geometries.

The point is further illustrated in figure 10 for a rubber-modified polypropylene in which K_c values in both tension and bending are shown for various thicknesses (Fernando & Williams 1980). The tension data have a strong thickness dependence, while the bending data are virtually constant until the thickness is less than that prescribed by the A.S.T.M. plane strain limit as shown. Thus, a plane strain value of $K_c \approx$ 4.0 MN/m$^{\frac{3}{2}}$ can be defined for this material but it is only true for a given set of circumstances, in this case low rates. Impact tests, for example, would suggest values of $K_c \approx$ 1.6 MN/m$^{\frac{3}{2}}$ for this material at high rates where, presumably, a micro-deformation mode is excluded.

Table 1 lists some collected values of K_c and G_c for some classes of non-metallic materials, together with average values for steel and an aluminium alloy. The groupings are rather coarse but they do give some indication of the spectrum of behaviour. K_c gives an indication of the strength of cracked components, while G_c determines the energy absorbing capacity of the material. The connecting factor is, of course, the modulus, E, also shown and ρ, the density, can be regarded as the price paid in terms of mass. As expected, metals show themselves as very high on all stiffness and strength scales, even when compared on a mass basis. The very low energy capacity of glass is clear as is the low stiffness but high energy capacity of rubber. Plastics show a compromise, and thermoplastics are reasonably good in all categories, especially when viewed on a comparative mass basis. The various composites listed are really compromises between the various basic materials and give some indication of why they have succeeded, although the very high strength composites are not listed.

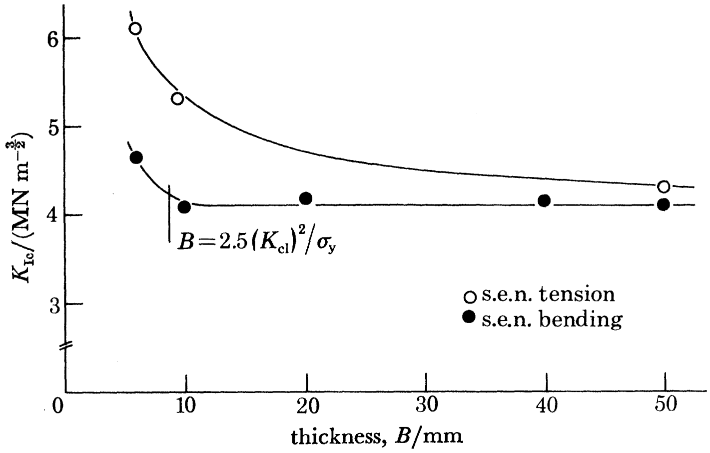

FIGURE 10. Fracture toughness as a function of thickness for polypropylene at −60 °C in tension and bending (Fernando & Williams 1980).

TABLE 1. SHORT-TERM LOADING FRACTURE TOUGHNESS FOR VARIOUS CLASSES OF MATERIALS: PLANE STRAIN VALUES

material	$K_c/(\mathrm{MN/m^{\frac{3}{2}}})$	$E/(\mathrm{GN/m^2})$	$G_c/(\mathrm{J/m^2})$	$\rho/(\mathrm{kg/m^3})$
thermoplastics	1–2	3	300–1300	1000
thermosets	0.6	3	120	1000
glass	0.7	70	7	2500
wood	0.5	2.1†	120	500
g.r.p. (glass (33 %) + thermosets, random)	7.0	7.0	7000	1500
glass reinforced thermoplastics (40 % glass)	4.0	6.3	2500	1500
chipboard (wood fibre reinforced thermosets, 10 % resin)	1.3	2.5	680	700
rubber	*ca* 0.1‡	0.002	3–10000	1000
steel	50–100	200	10–50000	8000
aluminium alloy	30	70	13000	2700

† Conversion factor for K_c to G_c (Sih *et al.* 1965).
‡ Computed as $\sqrt{(EG)}$, where E is the low strain modulus.

TIME DEPENDENCE

Many non-metallic materials show significant time dependence at normal operating temperatures because of their relatively high molecular mobility as compared with metals. Such behaviour is manifested in creep and relaxation, and is characterized by changing moduli with time or with a loss process, usually described in terms of a loss factor, $\tan\delta$. For materials in which the time dependence is not great, it is often found that log–log plots of the varying parameter with time are approximately linear so that the degree of time dependence can be designated by a parameter n from a relation of the form $E \propto t^{-n}$. This parameter is related to $\tan\delta$ by

$$\tan\delta = \tan(\tfrac{1}{2}\pi n).$$

The presence of time dependence means that the energy absorbing processes at the crack tip are also time dependent, and since the time scale for these processes must be inversely proportional to the crack speed, $\dot{a}$, then

$$E \propto \dot{a}^{n}.$$

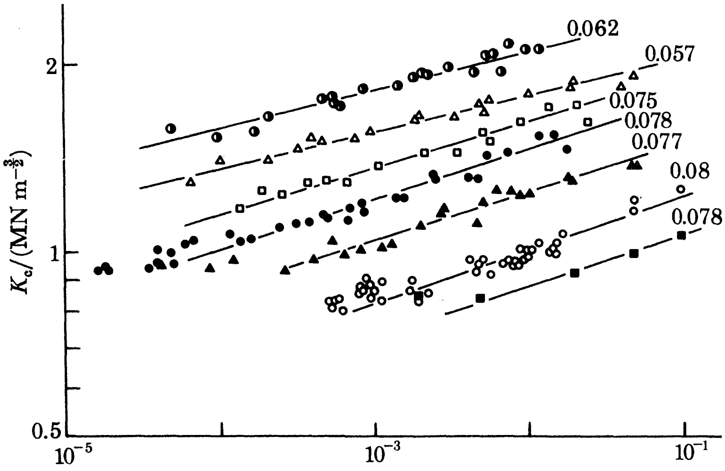

FIGURE 11. Crack growth in PMMA at various temperatures. (Data plotted on a logarithmic basis.) ■, +80 °C; ⊙, +60 °C; ▲, +40 °C; ●, +20 °C; ⊡, 0 °C; △, −20 °C; ◑, −40 °C. (Slopes are indicated at the right of each plot.) (Marshall & Williams 1975).

Thus, as $\dot{a}$ increases, so does E and related properties such as shear yield stress so that the fracture energy, which depends on these, tends to rise also. This results in stable crack growth in that K_c or G_c increase with speed. Thus cracks propagate slowly for very long periods, giving delayed fracture and phenomena often referred to as creep rupture or static fatigue.

Thermoplastics provide many examples of this since they frequently have viscoelastic loss processes operating within certain rate and temperature ranges, giving rise to $\tan\delta$ peaks and pronounced creep. That this is reflected in crack growth is illustrated in figure 11 (Marshall & Williams 1975), which shows $\lg K_c$ against $\lg \dot{a}$ for PMMA at several temperatures, where there is a pronounced $\tan\delta$ (β) peak in PMMA. The rising K_c with $\dot{a}$ is apparent and the slopes of the lines ($n \approx 0.08$) agree fairly well with measured $\tan\delta$ and creep behaviour. Such a relation between K_c and other properties may be deduced on the assumption of a characteristic crack tip distance which enables the local properties to be related to K_c and one such is the crack opening displacement, u, given by

$$u = \frac{G_c}{\sigma_y} = \frac{K_c^2}{\sigma_y E}, \tag{7}$$

where σ_y is the yield stress. Assuming that u is a constant at fracture and that $\sigma_y \propto E$ (a constant yield strain, e_y), we have:

$$K_c = \surd(u e_y) E \propto \dot{a}^{n}, \tag{8}$$

as observed. It is found that $u \approx 2$ µm and this has been confirmed by direct observation (Morgan & Ward 1977).

Very similar behaviour has been observed in crack growth in joints made with epoxy adhesives (Gledhill *et al.* 1978), and figure 12 shows $\lg K_c$ against $\lg \dot{a}$ for such a system, which is accurately described by $u \approx 0.5$ µm in these more brittle thermosetting materials; u may be regarded as the crack tip diameter and it has been shown that in a wide range of epoxy materials this parameter, the measure of notch tip sharpness, controls the onset of crack jumping, or stick-slip (Kinloch & Williams 1980). The idea of using the notch tip diameter, together with tensile properties, has also been used for slow crack growth in rubber (Greensmith 1960), and an example is given in figure 13 which shows $\lg T(\equiv G)$ against $\lg R(\equiv \dot{a})$ for direct observation and prediction with the use of $u \approx 480$ µm. The slope of this line is of a similar order to $\tan\delta$ but the correlation is complicated by the large strains involved, which affect $\tan\delta$. It is interesting to note that this work discusses the correlation of u and notch tip form with micro-processes, which is a concept now utilized for epoxies (Kinloch & Williams 1980) and in metals (Ritchie *et al.* 1973). Some data for slow crack growth in wood are shown in figure 14. As with epoxies, the time dependence is slight and, in addition, there is the inherent scatter of natural materials, but the data are sufficiently good to establish a line (Ewing & Williams 1979).

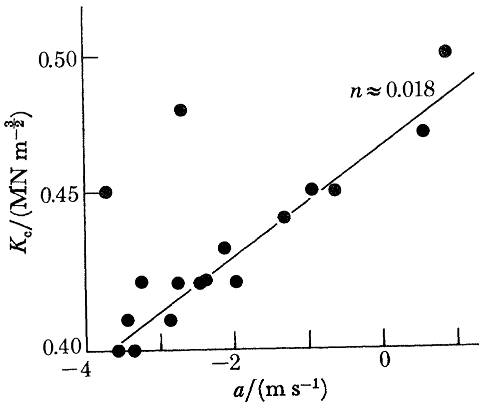

FIGURE 12. Crack growth in an epoxy resin adhesive (Gledhill *et al.* 1978).

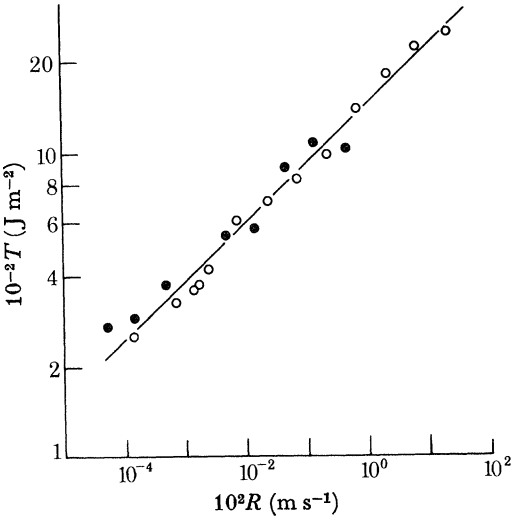

FIGURE 13. Comparisons of observed tear behaviour (○) and that calculated from tensile data (●) for SBR gum vulcanizate at +90 °C (from Greensmith 1960).

Data of this kind have been used to predict creep failures by putting $K = K_1\dot{a}^n$ and assuming an inherent crack size of a_0. If we take an infinite plate, then $Y^2 = \pi$, and (2) becomes

$$K_1\dot{a}^n = \sigma\sqrt{(\pi a)}.$$

The time to failure may be obtained by integrating this function via the relation

$$t = \frac{2}{\sigma^2\pi}\int_{K_0}^{K_{\mathrm{Ic}}} \frac{K\,\mathrm{d}K}{\dot{a}}, \tag{9}$$

where K_0 is the initiation value, and K_{Ic} the instability value of K, so that the life may be predicted for a given a_0, since $K_0 = \sigma\sqrt{(\pi a_0)}$. Some success with this type of approach has been reported for polymers (Young & Beaumont 1976), wood (Mindess *et al.* 1975) and glass (Evans 1973).

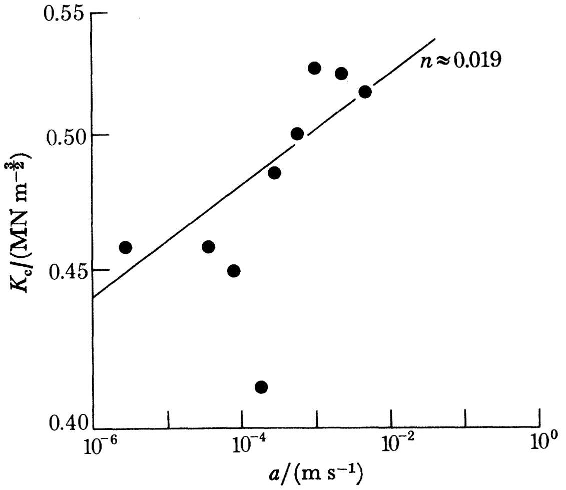

FIGURE 14. Slow crack growth in Scots pine (Ewing & Williams 1979).

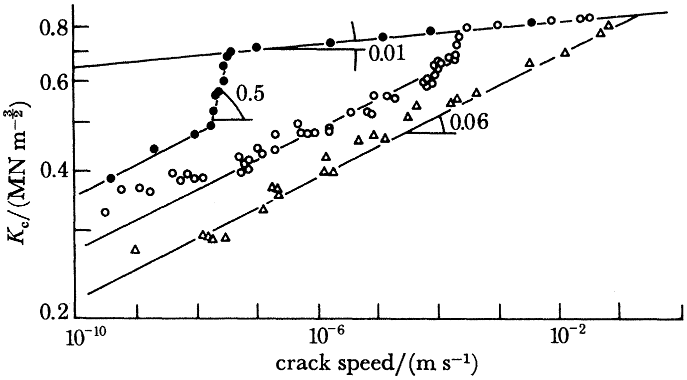

FIGURE 15. Crack growth data for inorganic glass in environments: ▲, water; ○, air (50 % r.h.); ●, dry paraffin (Weidmann 1973).

A final point in slow crack growth may be illustrated by reference to similar data in glass shown in figure 15. It has been shown that the yield stress in glass (determined by indentation) is time dependent and in air and water has an n of 0.06 (Marsh 1964) and in paraffin $n = 0.01$ (Gunasekera 1970). It can be seen that the crack growth data have an upper line with $n = 0.01$ and separate lines for air, water and paraffin with $n = 0.06$. It is well established that water decreases the crack tip stress in glass and this is reflected in the low K_c values at low speeds where water is present. At higher speeds and over most of the range for dry paraffin, the water cannot be transported quickly enough to the crack tip and so there is a common line, which represents the behaviour without water and reflects dry yielding behaviour. The different K levels represent the amount of water present and the lines show a time dependence appropriate to yielding in water. Again, a constant u concept works well (Weidmann 1973) and gives a dry fracture value of 2 nm. A good deal of data for ceramic materials in the form of K_{C}–$\dot{a}$, similar to those shown here, have been obtained (e.g. by Williams & Evans 1973; Weiderhorn 1967).

Similar data may be obtained for polymers in environments (Williams & Marshall 1975) and illustrate that slow crack growth with environments may be obtained at extremely low surface energies since chemical action contributes to the fracture process.

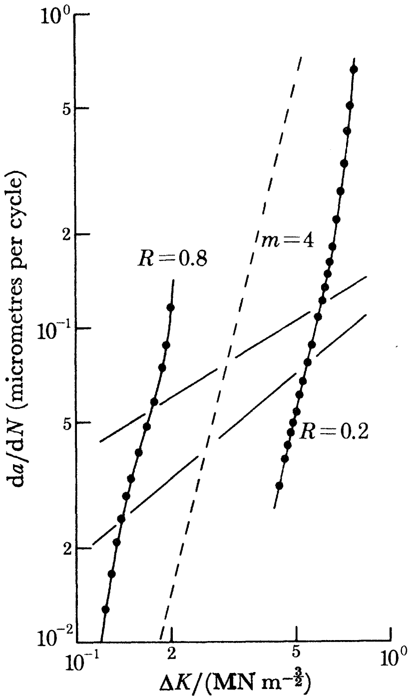

FIGURE 16. Fatigue crack growth data for PMMA at 20 °C and 10 Hz.

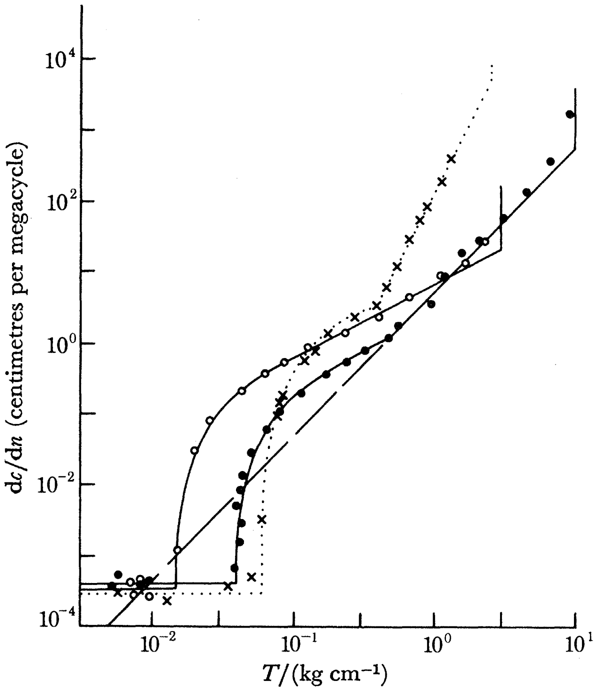

FIGURE 17. Full cut growth characteristics (logarithmic scales:) ●, vulcanizate A; ○, vulcanizate B; ×, vulcanizate C (SBR) (Lake & Lindley 1965).

FATIGUE

Fatigue is the process of crack growth under cyclic stress and results in crack growth at K levels below which single loading crack growth occurs. Since it is a crack growth phenomenon, then it is reasonable to suppose that the growth rate will be governed by K (or G) and so general relations of the form

$$\mathrm{d}a/\mathrm{d}N = F(K)$$

would be expected, where $\mathrm{d}a/\mathrm{d}N$ is the crack growth per cycle. At K levels where time dependent crack growth occurs, a growth rate may be deduced from $(\mathrm{d}a/\mathrm{d}t)(\mathrm{d}t/\mathrm{d}N)$, but fatigue occurs at K levels below where this happens.

The most widely used fatigue law was developed for metals by Paris & Erdogan (1963) and has the form

$$\mathrm{d}a/\mathrm{d}N = A\Delta K^m, \tag{10}$$

where A and m are constants, and ΔK is the range of K variation in the cycle. It was in general observed that $m \approx 4$ but that a range of values from 2 to 8 occurred. This representation of data has been generally successful in non-metals and there is a considerable volume of information on polymers (Manson & Hertzberg 1973) presented in this form. Figure 16 shows some fatigue crack growth data for PMMA at 20 °C and 10 Hz plotted on a log–log basis. The two lines are for $R = K_{\min}/K_{\max}$ ratios of 0.2 and 0.8, illustrating that the use of a simple ΔK in (9) is not adequate, but certainly for a given R value, $m = 4$ provides a good representation of the data and is sufficient for many design purposes. It can be seen, however, that even here there is evidence of changes in slope in sections of the curve and that these changes often coincide with fracture surface features corresponding to different fracture micromodes (Teh *et al.* 1979). The point is further illustrated by the data for rubber shown in figure 17, here plotted as $\lg T(\equiv G)$ against $\lg(\mathrm{d}a/\mathrm{d}N)$. At the higher T values, there is a slope of around 2 and the general law of $\mathrm{d}\dot{a}/\mathrm{d}N \propto T^2(\equiv K^4)$ was noted at a very early stage in rubber (Thomas 1958). At lower T values, the data change to a form of $\mathrm{d}a/\mathrm{d}N \propto (T-T_0)$, giving a clear fatigue limit. The constant $\mathrm{d}a/\mathrm{d}N$ at very low T values is due to ozone cracking and illustrates that even the very low fatigue limit can be further depressed by environmental effects.

Discussion

The basic point to be demonstrated here, namely that K_c and G_c may be used to characterize fracture in a wide range of non-metallic materials, has been amply confirmed. When plastic flow, or some other dissipative process, occurs then, as in metals, although a K_c may be measured for a given geometry, it is often dependent on the mode of loading. The test conditions can be arranged to give a consistent value by observing the rules for achieving plane strain conditions, but it is clear that such values are minima only for one set of conditions. In non-metals, plane strain values are often rate and temperature dependent, since micro-voiding at the crack tip can give small-scale plane stress and thus a local high degree of molecular mobility. Certainly, such values usually vary less than those with lower constraint, but it is very difficult to define an absolute minimum in any sense. This is further emphasized by environmental and fatigue effects, which can give crack growth down to energy values rapidly approaching zero.

If this point is accepted, then it is clear that a design philosophy based on using an absolute minimum is not realistic and we must replace the concept with that of a minimum for a prescribed set of conditions appropriate to the design. One may retain the plane strain requirement and then choose a set of conditions such as rate, temperature and environment, but here one is utilizing a risk factor by defining these. Although this may not be defined, it is inherent in the conditions chosen and one must then raise the question of why plane strain is used. Certainly, many materials used in practice are employed because they exhibit ductility, which is excluded in plane strain, and designs based on it are often economically unreasonable. In less critical applications, common in the use of non-metals, quite substantial risk levels are acceptable economically and it is difficult to define these quantitatively. Clearly, there is much scope for applying our knowledge of the basic fracture behaviour of materials to a detailed analysis of the statistics of design conditions. There is little work done in this field and it is urgently required; until it is available, design must rely on engineering judgement.

References (Williams)

Barrett, J. D. 1975 Paper presented at IUFRO Wood Engineering Meeting, Delft, Holland, 7–11 April.

Evans, A. G. 1973 *Int. J. Fract.* **9**, 267.

Ewing, P. D. & Williams, J. G. 1979*a* Paper presented at Third International Conference on the Mechanical Behaviour of Materials, Cambridge, England, 20–24 August.

Ewing, P. D. & Williams, J. G. 1979*b* *J. Mater. Sci.* **14**, 2959–2966.
Fernando, P. L. & Williams, J. G. 1980 *Polym. Engng Sci.* **20**, 215–220.
Gledhill, R. A., Kinloch, A. J., Yamini, S. & Young, R. J. 1978 *Polymer* **19**, 574–582.
Greensmith, H. W. 1960 *J. appl. Polym. Sci.* **3**, 183–193.
Greensmith, H. W. 1963 *J. Polym. Sci.* **10**, 291–318.
Griffith, A. A. 1920 *Phil. Trans. R. Soc. Lond.* A **221**, 163–197.
Gunasekera, S. P. 1970 M.Sc. thesis, University of Keele.
Irwin, G. R. 1948 *Fracturing of Metals.* Cleveland, Ohio: American Society of Metals.
Kinloch, A. J. & Williams, J. G. 1980 *J. Mater. Sci.* **15**, 987–996.
Lake, G. J. & Lindley, P. B. 1965 *J. appl. Polym. Sci.* **9**, 1233–1251.
Manson, J. A. & Hertzberg, R. W. 1973 *Fatigue failure in polymers.* (*C.R.C. Critical Reviews in Macromolecular Science.*)
Marsh, D. M. 1964 *Proc. R. Soc. Lond.* A **279**, 420.
Marshall, G. P., Coutts, L. H. & Williams, J. G. 1974 *J. Mater. Sci.* **9**, 1409–1419.
Mindess, S., Nadeau, J. S. & Barrett, J. D. 1975 *Wood Sci.* **8**, 389–396.
Morgan, G. P. & Ward, I. M. 1977 *Polymer* **18**, 87.
Newmann, L. V. & Williams, J. G. 1980 *J. Mater. Sci.* **20**, 572–578.
Orowan, E. 1955 *Weld. J.* **34**, 1575–1605.
Paris, P. C. & Erdogan, F. 1963 *J. Basic Engng* D **85**, 528.
Parvin, M. & Williams, J. G. 1975 *J. Mater. Sci.* **10**, 1883.
Ritchie, R. O., Knott, J. F. & Rice, J. R. 1973 *J. Mech. Phys. Solids* **21**, 395–410.
Rivlin, R. S. & Thomas, A. G. 1953 *J. Polym. Sci.* **10**, 291–318.
Schniewind, A. P. & Pozniak, R. A. 1971 *Engng Fract. Mech.* **2**, 223–233.
Sih, G. C., Paris, P. C. & Irwin, G. R. 1965 *Int. J. Fract. Mech.* **1**, 189–203.
Teh, J. W., White, J. R. & Andrews, E. H. 1979 *Polymer* **20**, 755–771.
Thomas, A. G. 1958 *J. Polym. Sci.* **31**, 467–480.
Turner, C. E. 1979 In *Post-yield fracture mechanics* (ed. D. G. H. Latzko), pp. 23–210. London: Applied Science Publishers.
Weidmann, G. W. 1973 Ph.D. thesis, University of Keele.
Wiederhorn, S. M. 1967 *J. Am. ceram. Soc.* **50**, 407–414.
Williams, D. P. & Evans, A. G. 1973 *J. Test. Evaluation* **1**, 264.
Williams, J. G. 1972 *Int. J. Fract. Mech.* **8**, 393.
Williams, J. G. & Marshall, G. P. 1975 *Proc. R. Soc. Lond.* A **342**, 55–77.
Young, R. J. & Beaumont, P. W. R. 1976 *Polymer* **17**, 717–722.

Discussion

T. J. PATRICK (*Department of Physics and Anatomy, University College London, U.K.*). The speaker's slide comparing fracture parameters for non-metallic materials included a glass. Are such data yet available for low-expansion glasses for astronomical telescopes, or for ceramics?

J. G. WILLIAMS. There are considerable data on the fracture of inorganic glasses and ceramics in the literature. Work published under the names of S. Wiederhorn and A. G. Evans is likely to be the most relevant.